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MATRIX METHOD OF PARALLEL DECOMPOSITION FOR MINIMIZATION OF SYMMETRIC BOOLEAN FUNCTIONS IN THE FORM OF EXTENDED POLYNOMIAL

A matrix method of parallel decomposition in order to minimize symmetric Boolean functions in orthogonal form of representation in the form of extended polynomial by modulus 2 has been developed. Symmetrical Boolean functions are characterized by the fact that they are not minimized in classical form of representation, but well – in the form of Zhegalkin polynomials. Compared to Zhegalkin polynomials, extended polynomials have better indicators of the complexity of implementing digital devices by total coefficient S_L *(1.49 times) and by total coefficient* S_{AD} *(2.37 times) due to a slight deterioration of the total coefficient* S_S *(deterioration of 1.293 times). The coefficient* S_S *is less important for the development of digital devices than the coefficients* S_L *and* S_{AD}*.*

Another advantage of using extended polynomials consists in the use of the idea of polarization of inputs of Boolean functions. Due to this, this method can be used as a powerful component of complete matrix method of parallel decomposition for obtaining a complex minimal form of Boolean functions, which has the best indicators of the complexity of digital blocks implementation due to a slight decrease in the speed of their work.

Unlike Zhegalkin polynomials having only one variant of the minimal form, an extended polynomial can have several minimal forms with the same complexity of implementation, that is essential for minimizing the systems of Boolean functions.

An essential feature of implementation of the method consists in the use of ready-made expanded matrices and tables of a complete list of conjunctive sets, which significantly accelerates the process of minimization in time

Key words: *symmetric Boolean function, minimization of symmetric Boolean functions, orthogonal form of representation, classical form of representation, polynomial form of Reed-Muller representation, Zhegalkin polynomial, extended polynomial of sum by modulus 2.*

Relevance of research. Minimization of Boolean functions (BF) with a large number of arguments is one of the key and time consuming steps in the process of synthesizing digital blocks (DB) of promising computer systems

The development of new minimization methods is intended to accelerate the minimization process for BF with a large number of arguments, to find optimal options for minimal forms, depending on the defining values of the complexity of the implementation. New methods are realized on the basis of the investigation of the internal structural structure of BF in terms of the phenomenon of decomposition of Boolean functions (1) and the genetic fractal nature of the internal structure of BF [2].

$$y = f(x_1, x_2, ..., x_n) = \overline{x_i} \wedge Q_{i0}(x_1, ..., x_{i-1}, x_{i+1}, ..., x_n) \cup x_i \wedge Q_{i1}(x_1, ..., x_{i-1}, x_{i+1}, ..., x_n) \quad (1)$$

As a result, an arbitrary BF is the result of a mutual composition of n pairs of maternal BFs obtained on the basis of a mutual non-contradiction. Each BF contains its own genetic code – complete information about all the maternal BFs on which it is based.

Empirical studies of genetic relationships between maternal and daughter BFs based on the properties of the decomposition phenomenon made it possible to construct a genealogy tree [2], which combines all BFs [2] within their complete set. The tree structure shows a clear relationship between the entire BF and the influence of the internal properties of the parent BF on a specific Boolean function.

Primary maternal BFs are two BF groups with one argument that defines BF properties in terms of minimization:

- Boolean function-constants (***BF-CONSTANTS***) – the logical "0" and the logical "1";
- ***SYMMETRIC BF***, Boolean functions with one argument containing one unit in their own number (BF with numbers 01 and 10)

Boolean functions ***BF-CONSTANTS*** are key when minimizing BF in the classical form of representation (CFR). It is precisely the presence of BF data in the genetic code of a specific BF, which is a necessary and sufficient condition for their minimization in the classical form of presentation. In fact, minimizing BF in the orthogonal representation form [1] for obtaining a result in the boundary form of the CPF is reduced to the search for ***BF-CONSTANTS*** [3].

A symmetric Boolean function (***SYMMETRIC BF***) is a function whose value does not depend on the permutation of its input bits, but depends only on the number of units at the input. The Boolean functions ***SYMMETRIC BF*** in the boundary form of the CFR are generally not minimized, but are perfectly minimized in another boundary form – in the polynomial form of the representation of Reed-Muller (RMFR).

The search for a minimal form of BF or in one of the boundary forms or in a consolidated complex form is determined in its genetic code by the ratio of maternal ***BF-CONSTANTS*** and ***SYMMETRIC BFs***. If ***BF-CONSTANTS*** or ***SYMMETRIC BFs*** are dominant, the most extreme form of representation is optimal. If the ***BF-CONSTANTS*** and the ***SYMMETRIC BFs*** are in parity – the optimal is the consolidated complex form of representation.

Analysis of recent research and publications. Analysis of recent research and publications. The problem of constructing an effective method of minimizing symmetric BF is devoted to a number of works, a simple list of which is quite solid. In these works the features of symmetric BF and ways of optimization of the process of minimization are considered. This problem is global and, as a result, has no simple solution. Zakrevsky's scientific group made a significant contribution to the development of this topic over the last few decades. The last publications of this group include works [4,5], in which the features of RMFR are studied. Considerable attention is paid to this subject at Kiev Polytechnic Institute [6], which describes the study of the process of minimizing of RMFR. In [7], an algorithm for minimizing systems of partially determined Reed-Muller polynomials is described. Sufficient attention to the Reed-Muller polynomial system is given in works [8, 9, 10].

Formulating the goals of the article. Symmetric Boolean functions are of particular interest in the development of computing digital blocks and blocks of coding systems. Considering their key importance, **the actual problem is** the construction of an effective method for minimizing symmetric BF in an orthogonal form of representation (ORFP) for a given group of Boolean functions that could independently be used to obtain a result in the boundary form – RMFP, or as part of the minimization to obtain the result in a consolidated complex presentation form, which is a mixture of KFR and RMFR.

The purpose of the paper is to develop an effective method for minimizing symmetric Boolean functions based on the polarization of their inputs in the form of an extended series.

Presenting main material. Taking into account the experience of constructing a matrix parallel decomposition method in the ORFP [3], the same ideas were used when developing a method for minimizing symmetric BF. In fact, the proposed method of minimization is an extension and weighty component of the complete matrix method of parallel decomposition on symmetric BF.

The essence of the method and the algorithm of minimization should be explained on a concrete example. Let BF contain 4 arguments and have a number $64\ 975_{10}$ (binary vector – $1111\ 1101\ 1100\ 1111_2$).

The minimization algorithm consists of the following steps:

1. Construction of a matrix for obtaining a result in the form of a polynomial in modulus 2 conjunctive sets of direct arguments. The matrix is square, has a dimension 2^n, where – n is the number of arguments in BF. The matrix contains four conditional parts – the left lower part is always zero, and the remaining 3 parts are identical with each other. The matrix is constructed on the basis of a formal primary matrix. The primary matrix is the matrix of 1 argument (2).

1	1
0	1

(2)

It is the basis for constructing the following matrices.

1	1	1	1	1	1	1	1
0	1	0	1	0	1	0	1
0	0	1	1	0	0	1	1
0	0	0	1	0	0	0	1
0	0	0	0	1	1	1	1
0	0	0	0	0	1	0	1
0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	1

(3).

When increasing the arguments in BF per unit, the size of the matrix is increased by 2 times. Each next matrix is filled in similarly – at first it is divided into 4 parts. In the lower left, all the elements fill with zeros, the remaining 3 parts – insert the previous matrix. For 3 arguments the matrix has the form (3):

For BF of 4 arguments, respectively, the matrix has a size 16 * 16 and a form (4):

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
0	0	0	0	0	1	0	1	0	0	0	0	0	1	0	1
0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1
0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

(4)

Each line of this matrix is a list of coefficients of the polynomial (5) of the corresponding line of the table of truth that describes this Boolean function. If the coefficient is "1", then the corresponding member is present, "0", then the corresponding member is absent.

$$=1\oplus x_1 \oplus x_2 \oplus x_2x_1 \oplus x_3 \oplus x_3x_1 \oplus x_3x_2 \oplus x_3x_2x_1 \oplus x_4 \oplus x_4x_1 \oplus x_4x_2 \oplus x_4x_2x_1 \oplus x_4x_3 \oplus x_4x_3x_1 \oplus x_4x_3x_2 \oplus x_4x_3x_2x_1 \quad (5)$$

2. Construction and computation of an expanded matrix to obtain a result in the form of a polynomial in modulus 2 conjunctive sets of direct arguments (6).

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	1
0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	1
0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	1
																0
																0
0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
																0
0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	1
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0	1	0	1	1	1	1	1	

(6)

When calculating, you need to create an expanded matrix by adding the column number BF to the right, and the bottom line of the final result. The rows opposite the "0" in the result column are to be deleted. After that, in each column, calculate the amount in modulus 2 and put the result into the corresponding cell of the result string.

The result string, the lower line of the expanded matrix, is the basis for writing polynomial conjugate sets of direct arguments. Having deleted in expression (5) the corresponding members with the coefficient "0", an intermediate result is obtained - the final polynomial of conjunctive sets of direct arguments. It consists of 9 terms and has the form (7).

$$y=f(x_1,x_2,x_3,x_4)=1\oplus x_3 \oplus x_3x_2 \oplus x_4x_1 \oplus x_4x_2x_1 \oplus x_4x_3 \oplus x_4x_3x_1 \oplus x_4x_3x_2 \oplus x_4x_3x_2x_1 \quad (7)$$

3. Construction of an expanded polynomial based on a polynomial (7). Based on the table of the list of all conjugate sets for BF (see Table 1), it is necessary to optimize the expression (7), replacing members with direct arguments by members of the extended series. In the table, the members of a row are sorted in order of increasing coefficients of implementation of $\boldsymbol{S_{AD}}$, $\boldsymbol{S_L}$ and $\boldsymbol{S_S}$. In this case there are groups of elements of the members of the extended series, having the same indicators of complexity of implementation. Therefore, it is necessary to consider the option when several solutions with the same complexity of implementation will be obtained on the basis of expression (7).

Table 1

A complete list of conjunctive sets for BF containing 4 arguments and their schedules on a row (5)

№ in order	Extension members	The presence of members of a row (5)															
		1	x_1	x_2	x_2x_1	x_3	x_3x_1	x_3x_2	$x_3x_2x_1$	x_4	x_4x_1	x_4x_2	$x_4x_2x_1$	x_4x_3	$x_4x_3x_1$	$x_4x_3x_2$	$x_4x_3x_2x_1$
1	$\overline{x_4}\cdot\overline{x_3}\cdot\overline{x_2}\cdot\overline{x_1}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	$\overline{x_4}\cdot\overline{x_3}\cdot\overline{x_2}\cdot x_1$	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
3	$\overline{x_4}\cdot\overline{x_3}\cdot x_2\cdot\overline{x_1}$	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
11	$x_4\cdot\overline{x_3}\cdot\overline{x_2}\cdot x_1$	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
18	$\overline{x_4}\cdot x_3\cdot\overline{x_2}$	0	0	0	0	1	0	1	0	0	0	0	0	1	0	1	0
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
79	$x_4\cdot x_3\cdot x_2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
80	$x_4\cdot x_3\cdot x_2\cdot x_1$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

As a result of the search, we have received the final answer (8), which contains a single result:

$$y = 1\oplus\overline{x_4}\cdot x_3\cdot\overline{x_2}\oplus x_4\cdot\overline{x_3}\cdot\overline{x_2}\cdot x_1 \qquad (8)$$

Investigation of complete sets L_2 of Boolean functions from 2 arguments, L_3 of Boolean functions from 3 arguments and L_4 from 4 arguments in terms of minimization by the indicated method showed a significant improvement in the complexity of the implementation of devices constructed on the basis of this method in comparison with Zhegalkin algebra (see Table 2). Due to a slight deterioration of the $\boldsymbol{S_S}$ – factor of the input bus, the $\boldsymbol{S_{AD}}$ coefficient – the total number of conjunctives in combinational scheme – is reduced significantly, and the $\boldsymbol{S_L}$ coefficient is the total number of conjunctures in combinational scheme.

Table 2

Changing integral characteristics of the structural complexity of chips on the complete set of *L(2)*, *L(3)* and *L(4)* Boolean functions

№ in order	The number of arguments in the boolean function	The value of the total complexity of implementation factors								
		S_S			S_L			S_{AD}		
		Polyn Zegalkina	Advanced polynom	% of the original value	Polyn Zegalkina	Advanced polynom	% of the original value	Polyn Zegalkina	Advanced polynom	% of the original value
1	2	*32*	*29*	*90,6*	*40*	*29*	*72,5*	*32*	*6*	*18,7*
2	3	*848*	*948*	*111,8*	*1 664*	*1 116*	*67,1*	*1 024*	*340*	*33,2*
3	4	*228 608*	*295 541*	*129,3*	*1 081 344*	*727 569*	*67,3*	*524 288*	*220 922*	*42,14*

Conclusions:

1. A matrix method of parallel decomposition is developed for minimizing the symmetric Boolean functions in the orthogonal representation form. This method can be used independently to obtain the minimal form of the Boolean function in the boundary form in the form of a polynomial in the amount of 2 conjugate sets of arguments and in the form of a complete matrix parallel decomposition method in conjunction with [3].
2. The results obtained by this method have a number of advantages compared with the results in Zhegalkin polynomial – a significant improvement in the complexity of the implementation of digital devices by coefficients S_L (1.49 times) and S_{AD} (2.37 times) due to a slight deterioration of the coefficient S_S (deterioration in 1,293 times)
3. An essential feature of the method is the use of already ready extended matrices and tables of a complete list of conjunctive sets, which significantly accelerates the process of minimization over time.

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МАТРИЧНИЙ МЕТОД ПАРАЛЕЛЬНОЇ ДЕКОМПОЗИЦІЇ ДЛЯ МІНІМІЗАЦІЇ СИМЕТРИЧНИХ БУЛЕВИХ ФУНКЦІЙ У ВИГЛЯДІ РОЗШИРЕНОГО ПОЛІНОМА

В роботі розроблено матричний метод паралельної декомпозиції для мінімізації симетричних булевих функцій в ортогональній формі представлення у вигляді розширеного полінома суми за модулем 2. Симетричні булеві функції характеризуються тим, що вони погано мінімізуються в класичній формі представлення, але добре – поліномами Жегалкіна. Результати, отримані цим методом, порівняно з результатами в поліномі Жегалкіна мають суттєве покращення показників складності реалізації цифрових пристроїв за сумарними коефіцієнтами S_L (в 1,49 разу) та S_{AD} (в 2,37 разу) за рахунок незначного погіршення сумарного коефіцієнта S_S (погіршення в 1,293 разу), що не є таким значущим при розробці таких цифрових пристроїв, як коефіцієнти S_L і S_{AD}. Також за рахунок поляризації входів булевих функцій цей метод може бути використано як один із складових чинників повного матричного методу паралельної декомпозиції для отримання комплексної мінімальної форми булевих функцій, що має кращі показники складності реалізації, ніж класичні форми представлення булевих функцій. Цей метод дає можливість отримувати для булевих функцій кілька результатів з однаковими показниками складності реалізації, що є суттєвим при мінімізації систем булевих функцій. Суттєвою особливістю методу є застосування вже готових розширених матриць і таблиць повного переліку кон'юнктивних наборів, що суттєво прискорює процес мінімізації в часі.

Ключові слова: *симетрична булева функція, мінімізація симетричних булевих функцій, ортогональна форма представлення, класична форма представлення, поліноміальна форма представлення Ріда-Мюллера, поліном Жегалкіна, розширений поліном суми за модулем 2.*

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