METHOD OF CONSTRUCTING A MATHEMATICAL MODEL OF WEAR-RESISTANCE IN POTENTIOMETER SENSORS

This article highlights the method of constructing a mathematical model of wear-resistance in potentiometer sensors. Depreciation process of working surfaces is so complex that by this time there are no reliable methods of its forecasting. Tribological processes taking place during potentiometer sensor performance are complicated by electric current influence that flows through the contact surface and accelerates the process of contact surfaces wear. As a result, the mathematical model is offered, that considers the simultaneous action of tribological elastic contact, plastic deformation, microwear and electrical erosion with constant number of performance cycles. Wear-resistance is represented as the magnitude of change in volume of the worn material depending on the shape and material of contact surfaces, contact pressure and magnitude of electric current. The model is represented in the form of dependence, interpreted by the direct equation, greatly simplifies the forecasting of wear-resistance on the basis of experimental data. Mathematical model, obtained by adding bulk models of mechanical and current wear can be used to predict durability in the design and operation of potentiometer sensors, wire potentiometers and other devices that have a similar structure and work under the current.

Keywords: sliding contact, wire potentiometer, tribological processes, capacious erosion, reliability of work.

Introduction. Potentiometer sensors are designed to convert a mechanical movement into an electrical signal by changing the active resistance of electrical circuit. Signal of the engine position in the tension meter is being removed, as from the voltage divider, after the power supply to the sensor. Potentiometer sensors are used mainly as sensors of linear and angular displacements. Potentiometer sensor is designed for the following purposes: control and measurement of mechanism displacements, working bodies of machinery and other objects; feedback segment in robotics and automation systems; defining distances to objects; testing in laboratories, mechanism performance control. For example, these sensors enable to set control over the position of latches, valves, vane on the weather station, antennas, cutting tools and much more [1, 2, 3]. They can also be used to transmit the indicators of non-electrical metering devices, such as manometers, liquid level meters, to transmit the notification of aeroplane chassis, etc [1, 3].

Potentiometer sensor is a potentiometer of the changeable resistance itself, which can be switched on by following the scheme of circuit voltage divider (Figure 1) [1]. When moving a contact under the influence of controlled magni-
In automatic systems, the engine can be mechanically connected to any device (valve, steering wheel, receiver, cutting tool, etc.), location of the latter should be measured and passed in the form of an electric signal [3]. The effort, under the action of which the engine moves, is quite big in this case. Therefore, to ensure a reliable contact between engine and winding, you should have a pretty-strong force pressing the engine. The presence of a sliding contact reduces the reliability of potentiometer sensor and remains its basic disadvantage.

Structurally potentiometer sensor (Figure 2) consists of a frame 1, which is wrapped by one layer of the wiring 2 from the thin wire [2]. The engine (brush) slips 3 on the twisted coil, mechanically linked to the object and the displacement of which should be measured. The coil is made of isolated wire, and the path, on which the engine slips, is eliminated from isolation beforehand.

![Figure 2 – Line diagram of potentiometric sensor for measuring linear movement](image)

Resolution defines the maximum possible accuracy of potentiometric sensor performance. It can be improved by increasing the number of turns \( n \). It becomes possible via lengthening the wound part of the potentiometer \( L \) (at a given diameter of the wire), or reducing the cross section of the wire. Diameter reduction of the wire leads to technological difficulties in the manufacturing of windings, but, what is more important, reduces the reliability of the potentiometer, as mechanical strength of the winding is deteriorating and it is erased more quickly.

To ensure the reliability of the performance of sliding contacts in potentiometer sensors the requirements for their physical, mechanical and tribotechnical properties are set. Particular difficulty in solving tribotechnical problems in sliding contacts is an action of an electric current, which leads to a strengthening of friction and wear [4-7].

Classical wear theory considers the speed of material removal as a function of sliding speed, hardness of the material, applied load and the like-

likelihood that the material will create a part of wear in a given contact situation [8-12].

The wear mode, typical for the end seals of the boundary friction, is most fully described by I. V. Kragelsky who worked out molecular-mechanical (adhesion-deformation) theory of friction and fatigue wear [11, 12]. According to this theory friction is caused, on the one hand, by the deformation of the material, which leads to a violation of the whole (elastic or plastic imprint), on the other hand, by overcoming the molecular (adhesion) connections in the contact area.

The process of working surface wear is so complex, that by this time there are no reliable methods of its forecasting [6, 9, 13, 14]. Even for the same materials, the intensity of wear may vary in several orders while the operating mode is changed: when changing the pressure of the sealing fluid, angular velocity, temperature, axial and angular vibrations. So far, the estimation of wear indicators is based on the operation experience and high degree of authenticity can’t be demanded from them. The most reasonable and acceptable for engineering calculations formulae for wear intensity are given in the fundamental guide [1], but special physical and mechanical characteristics of these formulae (curve frictional fatigue setting; correction coefficient to the number of cycles corresponding to the separation of wear parts; coefficient characterizing the tense condition on the spot of contact, etc.) are partially systematized only for some of the most widespread constructional materials in dry friction conditions and do not take into account the influence of electric current.

In a number of works [7, 13] dependencies of wear rate from the contact pressure and the slip rate are experimentally defined. The Archard formula [8, 9, 15] contains parameters such as surface hardness, defined by the indentation method, the wear coefficient, which must be determined experimentally for each combination of material in the contacting pair and the seal liquid, and also for the certain operation conditions, including the friction mode, temperature, presence of vibrations, abrasive particles, etc.

It was Holm who proposed to take into account the impact of current on the value of mechanical wear in coal brushes of electric cars [6, 7, 9].

There are four dominant methodologies in this field – energy balance, mass balance, strain analysis, and contact mechanics approach [8, 9]. There is currently no direct connection or agreement between various models that can be found in
the literature, and formal standardization as such in this area does not exist, which creates difficulties for both researchers and engineers who are trying to fight the effects of wear in its various forms [14, 15].

The aim of the study is to obtain wear mathematical model of contact pair of slip under current load by combining and improving the known mathematical approaches, taking into account the design features of wire potentiometers and potentiometer sensors.

Presentation of the main material in the research. Structurally potentiometer sensors are similar to wire potentiometers. Material of current-collection element of the slider, as well as the material of the resistive element, should be resistant against electrical erosion and corrosion, easily handled, possess properties that prevent the welding of contacts, have high heat and electrical conductivity, high durability together with the selected resistive wire, small and stable in time joint resistance.

The proposed estimation methodology of the contact pair of slipping under the current with constant development cycles N is to represent the total volume of wear as a sum of two components [16]:

\[ V = V_m + V_t, \]

where \( V_m \) is a mechanical wear under current, equal to zero; \( V_t \) is an additional wear, caused by the passage of current through the contact, i.e. current wear.

Taking into account that the wire, wound on a potentiometer frame, represents a cylinder with the radius \( R \), and a sliding contact can be presented as a sphere of radius \( R \), the result of the interaction is presented in Figure 3 as a wear scar in the pair of a cylinder-sphere with the unchanged spherical radius \( R \).

Mechanical processes that occur as a result of contact movement affect the dependence of the transition resistance on the value of contact pressures and stresses – the predominant type of deformation processes in the contact material. The values of shock and vibration loads associated with movement, the speed of movement of the current collector, the coefficient of friction, and other mechanical factors can be represented as an energy model.

Mechanical energy of a single junction expended during friction and wear depends on the value of the maximum tangential stresses \( \tau \) or contact pressure \( P \) and the length of the junction path \( l \), i.e. \( W = f_1(\tau, l) \), or \( W = f_2(P, l) \).

Depending on the magnitude of tangential stresses and the state of the surfaces of the mating bodies, all three types of deformation processes can occur at the point of contact: elastic displacement, removal and micro-cutting due to wear particles.

In elastic contact, wear occurs due to long-term friction fatigue, and in plastic deformation due to crumpling forces, so the amount of wear in both cases will be proportional to the friction energy \( W_f \), but with different values of the proportionality coefficients [16].

The volume \( V_m \) of mechanically worn material at an elastic contact will be proportional to the friction power \( W_f \), and at the microcutting of the material is proportional to the energy of the shear force \( W_s \). To determine the wear patterns of contact pairs cylinder-sphere at \( W = \text{const} \) (for "dry contact pairs"), the volume of worn material will be determined by the generalized pattern of wear to height \( H \) or to chord \( T_r \).

\[ R \quad \text{radius sphere}; \quad r \quad \text{radius cylinder}; \quad H \quad \text{height wear}; \quad T_w \quad \text{chord of wear along the length of the cylinder}; \quad T_b \quad \text{chord wear}; \quad Q_b \quad \text{area along the length of the cylinder}; \quad Q \quad \text{area of the cylinder wear} \]

**Figure 3 – Wear scar in the pair of a cylinder-sphere with unchanged spherical radius \( R \)**

According to the notations taken in Figure 3, at constant geometric dimensions of contact \((4 \cdot H_m \to \infty)\), the volume of worn material of cylindrical wire in the transverse movement of spherical contact will be:
Let’s indicate the magnitude of volume $V$ through the controlled wear parameter $T_b$

$$Q_b = \frac{T_b^3}{16 \cdot r};$$

$$H = \frac{T_b^2}{8 \cdot r} = \frac{T_a^2}{8 \cdot R},$$

and here we get:

$$T_a = T_b \cdot \sqrt{\frac{R}{r}}.$$  \hspace{1cm} (3)

After the value substitution $Q_b$ and $T_a$ into (2) we get:

$$V = \frac{T_b^3}{48 \cdot r} \cdot \sqrt{\frac{R}{r}} = C_V \cdot T_b^4$$  \hspace{1cm} (4)

and

$$dV = \frac{T_b^3}{12 \cdot r} \cdot \frac{R}{r} dT_b = 4 \cdot C_V \cdot T_b^3 dT_b,$$ \hspace{1cm} (5)

where $C_V = \frac{1}{48 \cdot r} \cdot \sqrt{\frac{R}{r}}$.

The power of friction forces $W_f$ and cut $W_b$ at a single transition in the $dT_b$ path will be equal to [16]:

$$dW_f = K_f \cdot P_i dT_b,$$ \hspace{1cm} (6)

$$dW_b = \tau dQ dT_b,$$ \hspace{1cm} (7)

where $\tau$ is a yield limit per slice; $K_f$ is a coefficient of friction; $Q$ is a cross-section of the track at a single junction.

After integration within the wear boundaries $T_b$ of one transition and considering the performance within $N$ transitions, we get:

$$W_{f1} = K_f \cdot P_i \cdot T_b \cdot N,$$ \hspace{1cm} (8)

$$W_{b1} = \frac{\tau \cdot P_i \cdot T_b^4 \cdot N}{64 \cdot r} \cdot \sqrt{\frac{R}{r}}.$$ \hspace{1cm} (9)

Having considered $T - \tau \sim P_k^\frac{1}{3}$ and $W \sim V - T^4$, we obtain

$$W_{f2} \sim \tau^4 \cdot N \sim P_k^\frac{4}{3} \cdot N,$$ 

and

$$W_{b2} \sim \tau^4 \cdot N \sim P_k^\frac{4}{3} \cdot N$$

For the equally possible effect of the friction forces and shear we have:

$$V = K_v \cdot \tau^2 \cdot N,$$ \hspace{1cm} (12)

hence the wear pattern will be

$$T_b \sim \tau^3 \cdot N^4 \sim P_k^\frac{5}{4} \cdot N^\frac{5}{4},$$ \hspace{1cm} (13)

that corresponds to the accepted at present empirical engineering wear model [10, 18].

When moving the engine (brush) along the coils of potentiometric sensor winding, there is a sharp to contacts are opened under current. When the contacts are opened under current, there is a sharp increase in transient resistance and voltage drop on them, which leads to an increase in the temperature of contacted protrusions, up to melting point of contact material. At the first moment of contact opening is formed between them a bridge of molten metal contacts and metal contacts themselves, which in the further divergence of the contacts will not thin in the middle, but closer to positive electrode, where, at last, will be interrupted. This phenomenon is similar to electrolysis. This process causes the transfer of metal from one contact to another. This phenomenon is called bridging the erosion of the contacts. As a result of erosion, the microgeometry of the contact surfaces changes, which leads to increased mechanical wear of the contact surfaces. If the voltages and currents in the open circuit are lower than certain values for specific contact materials (for example, for silver $U < 12 V$, $I < 0.4 A$), bridge erosion will be the main type of electrical wear of the contacts. This is highly undesirable, since this mode of operation dramatically reduces the resource of normal operation of contacts [19].

Switching by contacts of low-power circuits with active load is accompanied by erosion phenomena caused by the formation of molten bridges in the contact zone at certain moments of contact movement. When switching current with breaking contacts, there are time intervals when such a small area contacts that the current density in it reaches values sufficient for melting and subse-
quent evaporation of the contact material. The liquid metal switches the inter-contact gap, forming a molten conductive bridge.

Bridges can occur both when closing and opening contacts. It is generally accepted that bridges on closed contacts arise due to the pulling of softened contact material heated by electrostatic radiation currents with sufficiently small inter-contact gaps by electrostatic field forces [7].

The maximum length of the bridge at the time of its explosion by current is from the ratio [7]

\[ S_{ex} = \nu_{m} \cdot \tau_{ex}, \]

where \( \nu_{m} \) is an average speed of opening the contacts of polarized relays, determined for each relay at the specified supply and control voltages, and \( \tau_{ex} \) is a time interval from the beginning of disconnection of the contacts to the moment of explosion of the bridge, determined by the voltage waveforms on the bridge.

We will define the value of current wear. Contacts, which dial currents, less than minimum current of the rod at voltages less than 300 V, are called low powerful. In this case, the erosion of the contacts occurs as a result of the formation of bridges and pulse low-voltage discharges. This transfer is called a thin transfer. For the bulk positive thin metal transfer of a single cycle we have a dependency of the type [7]:

\[ V_{f} = a \cdot I^{\alpha}, \]

where \( a \) is a coefficient of current impact on the wear that depends on the contact material; \( \alpha \) is an indicator of impact level of the current onto the wear.

In the vast majority of cases, the direction of the transfer is positive, that is, a crater appears on the anode, and a peak is on the cathode.

Total wear volume formula can be expressed via the relative value of the lateral wear:

\[ V_{z} = V_{m} \cdot \left( 1 + \frac{V_{f}}{V_{m}} \right), \]

or

\[ V_{z} = V_{m} \cdot \left( 1 + K_{f} \cdot I^{\alpha} \right), \]

where \( K_{f} = \frac{a}{V_{m}} \) is a relative coefficient of the current load.

Considering the wear pattern \( T \) of the contact pair of the cylinder-sphere with (12) and (13) in the general case, the volume of wear can be represented by the equation:

\[ V = m \cdot T^{4}. \]

where \( m \) is proportion coefficient.

Then

\[ \frac{V_{z}}{V_{m}} = \left( \frac{T_{z}}{T_{m}} \right)^{4}. \]

Considering \( t = \frac{T_{z}}{T_{m}} \), and taking into account (16) we get:

\[ t = (1 + K_{f} \cdot I^{\alpha})^{\frac{1}{4}}. \]

In the equation (17), regarding the wear of the contact pair cylinder-sphere coefficient \( K_{f} \) and degree indicator \( \alpha \) must be determined according to the research data. It is convenient to use the equation

\[ \lg (t^{4} - 1) = \lg K_{f} + \alpha \cdot \lg I, \]

obtained after taking the logarithm of the equation (17). The dependency (18) is expressed by the equation of the line, where \( \lg K_{f} \) is the initial coordinate and the indicator of degree \( \alpha \) is equal to the tangent of the straight line.

For approximate calculations, for \( K_{f} \cdot I^{\alpha} < 0.1 \) we can decompose equation (17) into a Maclaurin series, limiting it to two terms:

\[ t \approx 1 + \frac{K_{f}}{4} \cdot I^{\alpha} \approx 1 + K_{f} \cdot I^{\alpha}. \]

In turn, equation (19) can be represented as

\[ t \approx e^{K_{f} \cdot I^{\alpha}}. \]

In equation (20), the coefficients \( K_{f} \) and \( \alpha \) can be determined graphically, for which, by performing double logarithm, we obtain the equation of the line

\[ \ln^{2} t = \ln K_{f} + \alpha \cdot \ln I. \]

This approach simplifies the determination of coefficients and expands the scope of the model (17), but reduces the accuracy of forecasting.

**Conclusions.** As a result of the research, the mathematical model of potentiometer sensor wear-resistance has been developed considering
current loads. Given that the model (17) obtained by adding classical models (6), (7) and (14) and simplified by combining similar parameters, the accuracy of calculations for the second use will remain the same as for classical models, but the number of calculations will be reduced. It also reduces the number of experiments needed to determine unknown constants. The use of a single controlled wear parameter \( t \) is more convenient for predicting the wear resistance of potentiometric sensors by identification method. Presentation of the model in the form of the dependence, which is expressed by the equation of the line, greatly simplifies the forecasting of wear-resistance on the basis of experimental data, which is the main advantage of the obtained model. Developed mathematical model can be used for the prediction of wear resistance in the design and operation of potentiometer sensors, wire potentiometers and other devices, that have a similar structure and operate under the current.

To predict the wear resistance of potentiometric sensors, it is recommended to use an identification method that allows to get a model of changes in the chord of the wear trace of the wire \( T \) as the main parameter depending on the impact of the most significant operating factors: contact pressure \( P_c \), the number of cycles of working life \( N \) and switching current \( I \) with other constant factors (constant value of the path and speed of movement of the contact under normal environmental conditions). The choice of these parameters can be carried out both based on the materials of existing studies and on the results of experiments.

References

Список використаних джерел

МЕТОД ПОБУДОВИ МАТЕМАТИЧНОЙ МОДЕЛИ ЗНОСОСТОЙКОСТИ ПОТЕНЦИОМЕТРИЧНЫХ ДАТЧИКОВ

В статье рассмотривается метод построения математической модели зносостойкости потенциометричных датчиков. Физические процессы, которые в них происходят при работе потенциометричных датчиков, имеют два основных компонента: механический контакт и перенос энергии трения. Влияние этих процессов можно разделить на три основные составляющие: контактную деформацию, контактное трение и трение в точках контакта. Эти процессы зависят от величины контактного давления, уровня деформации, коэффициента трения и других факторов. Математическая модель, которая может быть представлена как сумма моделей, учитывающих эти факторы, может быть использована для прогнозирования зносостойкости потенциометричных датчиков и других аналогичных устройств.

Ключевые слова: ковзальный контакт, датчик потенциометр, трибологические процессы, надежность работы.